Inspiring Greatness

INDEPENDENCE SCHOOL DISTRICT

Math Virtual Learning

AP Stats

Difference of Means

April 24th, 2020

Lesson: April 24th, 2020

Objectives:

1. **Students will describe the characteristics of the sampling distribution** $\bar{x}_1 - \bar{x}_2$
2. **Students will perform a two-sample *t* test to compare means**
3. **Students will construct & interpret a two-sample *t* interval to compare two means**

The Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

Choose an SRS of size n_1 from Population 1 with mean μ_1 and standard deviation σ_1 and an independent SRS of size n_2 from Population 2 with mean μ_2 and standard deviation σ_2.

- **Shape:** When the population distributions are Normal, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is Normal. In other cases, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ will be approximately Normal if the sample sizes are large enough ($n_1 \geq 30$ and $n_2 \geq 30$).
- **Center:** The mean of the sampling distribution is $\mu_1 - \mu_2$. That is, the difference in sample means is an unbiased estimator of the difference in population means.

The Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

- **Spread:** The standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

*****Important Note:** When performing a significance test or constructing a confidence interval, the *population* standard deviations σ_1 & σ_2 are not known.
So, instead of using this formula for the standard deviation of the sampling distribution, you will use the **standard error** formula which is the same except it uses the standard deviations from the ***samples***.

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

as long as each sample is no more than 10% of its population (the 10% condition).

Objective 1 Example:

Researchers are interested in studying the effect of sleep on exam performance. Suppose the population of individuals who get at least 8 hours of sleep prior to an exam score an average of 96 points on the exam with a standard deviation of 18 points. The population of individuals who get less than 8 hours of sleep score an average of 72 points with a standard deviation of 9.4 points. Suppose 40 individuals are randomly sampled from each population.

a) Describe the shape, center, and spread of the sampling distribution of $\bar{x}_1 - \bar{x}_2$

Shape: each sample consists of 40 people so $n_1 \geq 30$ and $n_2 \geq 30$
The Central Limit Theorem says the sampling distribution is ~normal because each sample is large enough.

Center: $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 = 96 - 72 = 24$ (mean diff. in scores is 24 pts.)

Spread: $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{18^2}{40} + \frac{9.4^2}{40}} \approx 3.21$ Assuming each population has at least $40(10) = 400$ people

Objective 1 Example:

b) Find the probability of observing a difference in sample means of 2 points or more from the two samples. Show your work.

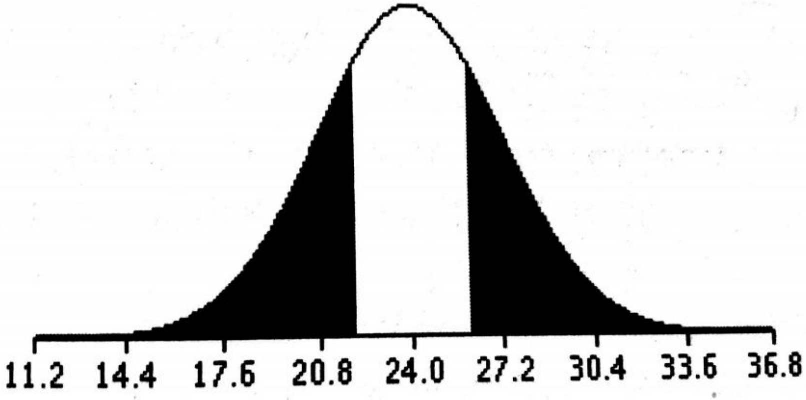
P(diff. of means is 2 or more points) = $P(\bar{x}_1 - \bar{x}_2 \le 22)$ and $P(\bar{x}_1 - \bar{x}_2 \ge 26)$

$Z = \dfrac{22-24}{3.21} \approx -.62$

$Z = \dfrac{26-24}{3.21} \approx .62$

normalcdf$(-10, -.62) \approx .268$

$2(.268) \approx .54$



Two-Sample *t* Test (***Significance tests for $\mu_1 - \mu_2$***)

1) STATE the parameter of interest and the hypotheses you would like to test.

The null hypothesis usually states that the parameters are equal while the alternative hypothesis is that one mean is greater than, less than, or not equal to the other:

H_0: $\mu_1 = \mu_2$

H_a: $\mu_1 > \mu_2$ OR $\mu_1 < \mu_2$ OR $\mu_1 \neq \mu_2$

State the significance level.

2) PLAN: Choose the appropriate inference method and check the conditions.

We must check the Random, Normal, and Independent conditions. Verify that the data come from random samples or the groups in a randomized experiment. To ensure that the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is at least approximately Normal, check that the population distributions are Normal OR n_1 and n_2 are both at least 30. If sampling is done without replacement, check that the populations are at least 10 times as large as the samples.

Two-Sample *t* Test (***Significance tests for $\mu_1 - \mu_2$***)

3) DO: If conditions are met, calculate a test statistic and *P*-value.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

and find the *P*-value by calculating the probability of observing a *t* statistic at least this extreme in the direction of the alternative hypothesis. Use the *t* distribution with df = smaller of $n_1 - 1$ and $n_2 - 1$ OR given by technology.

4) Conclude by interpreting the results of your calculations in the context of the problem.

If the *P*-value is smaller than the stated significance level, you have significant evidence to reject the null hypothesis. If it is larger than or equal to the significance level, then you fail to reject the null hypothesis.

Objective 2 Example:

Do boys have better short term memory than girls? A random sample of 200 boys and 150 girls was administered a short term memory test. The average score for boys was 48.9 with standard deviation 12.96. The girls had an average score of 48.4 with standard deviation 11.85. Is there significant evidence at the 5% level to suggest boys have better short term memory than girls? Note: higher test scores indicate better short term memory.

State: We will perform a 2-Sample t test for the difference in mean memory scores for boys + girls

$H_0: \mu_B = \mu_G$ or $\mu_B - \mu_G = 0$ $\alpha = .05$

$H_a: \mu_B > \mu_G$ or $\mu_B - \mu_G > 0$

Plan: CLT states the sampling distribution of the differences is ~Normal since $n_B = 200$ and $n_G = 150$, both greater than 30.

Objective 2 Example:

Plan Continued: Random sample of boys + girls was taken. We can assume at least 2000 boys + 1500 girls in the population.

diff. in the H_0

Do: $$t = \frac{(48.9 - 48.4) - 0}{\sqrt{\frac{12.96^2}{200} + \frac{11.85^2}{150}}} \approx .375$$

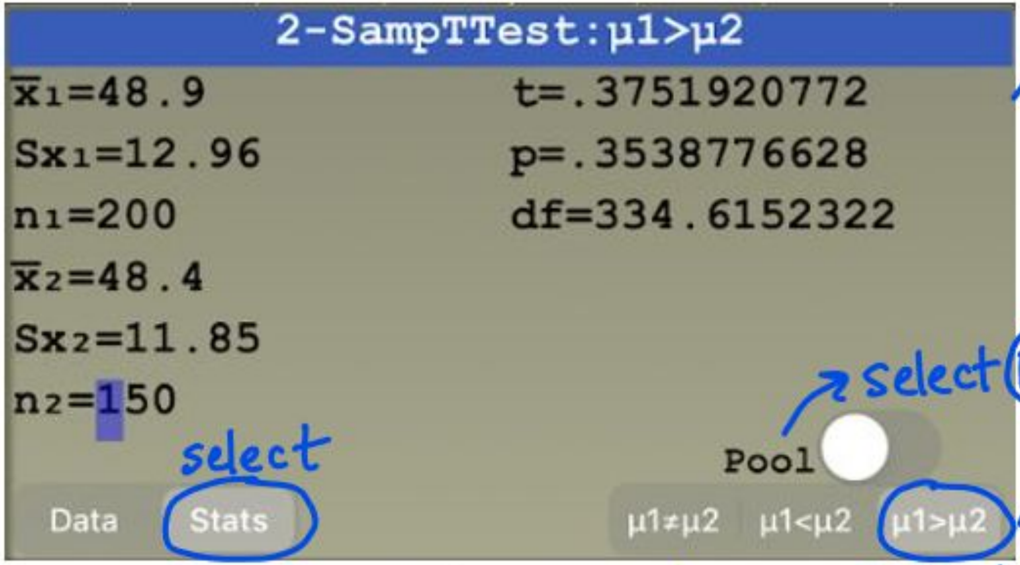
Using a 2-SampTTest in my calculator, I get a p-value $\approx .35$ and $df = 334.62$.

See directions on next slide.

Objective 2 Example: Technology

$\boxed{\text{STAT}} \rightarrow$ Tests $\rightarrow$ 2-SampTTest

The formatting is a little diff. on a regular TI-84 (this was done on a phone app)



select

select No for Pooled

← matches the H_a

Conclude: Since the p-value .35 is bigger than $\alpha = .05$, we fail to reject the H_0 + cannot conclude the H_a. We do not have evidence to suggest that boys have better short-term memory than girls.

Two Sample t interval

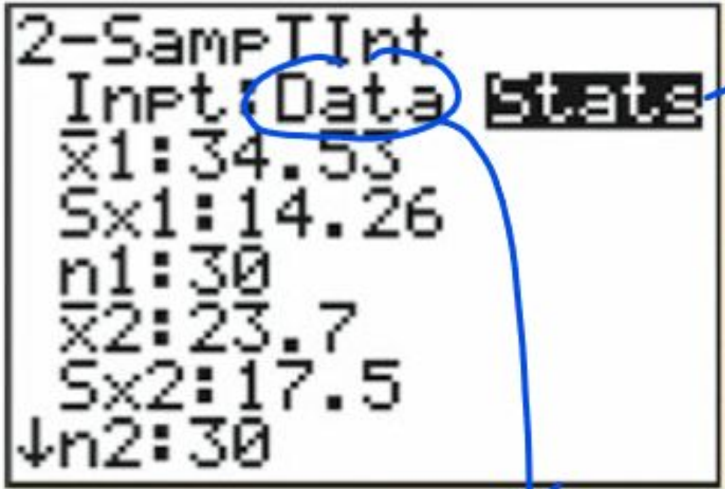
- **State** the parameters of interest and the confidence level you will be using to estimate the difference.
- **Plan:** Indicate the type of confidence interval you are constructing and verify that the conditions for Random, Normal, and Independent are satisfied for the two samples.
- **Do** the actual construction of the interval using the following formula:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

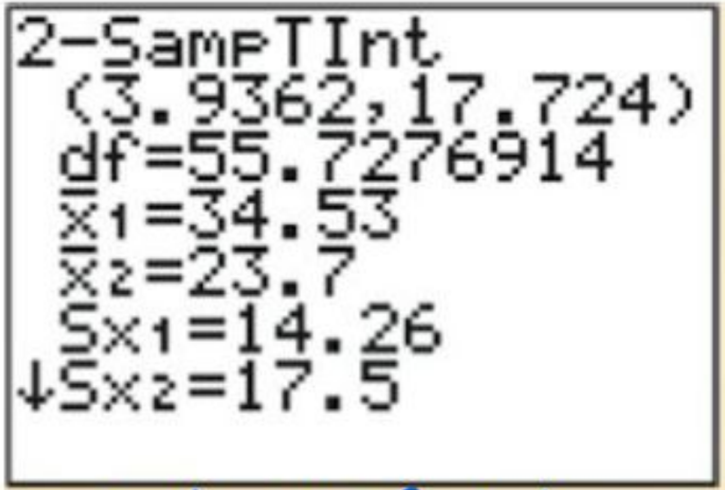
  where t^* is the critical value for the t distribution curve having df = smaller of $n_1 - 1$ and $n_2 - 1$ OR given by technology with area C between $-t^*$ and t^*.
- **Conclude** by interpreting the interval in the context of the problem.

Two Sample *t* interval: Using Technology

Press STAT → Tests → 2-SampTInt



select if you know the stats for both data sets



If you are given the actual sets of data then select "Data". Make sure the data is in L1 and L2.

Objective 3 Example:

Do boys have better short term memory than girls? A random sample of 200 boys and 150 girls was administered a short term memory test. The average score for boys was 48.9 with standard deviation 12.96. The girls had an average score of 48.4 with standard deviation 11.85.

Construct a two-sample t interval to estimate the true difference between boys and girls short term memory.

State: We want to estimate $\mu_B - \mu_G$, the true mean difference between boys + girls short term memory scores. Confidence Level = 95%

Plan: Same as previous example.

Do: In my calc.: 2-SampTInt

$\bar{x}_1 = 48.9$ $\bar{x}_2 = 48.4$ C-Level = .95

$S_{x_1} = 12.96$ $S_{x_2} = 11.85$ Pooled: No

$n_1 = 200$ $n_2 = 150$

Interval = $(-2.121, 3.1214)$ df = 334.62

Objective 3 Example Continued

Conclude: I am 95% confident that the true mean difference between boys + girls short term memory scores is captured in the interval −2.121 to 3.1214 points.

↓ Notice this includes 0 (the H_0 in the previous problem). So the true diff. in scores could be 0.

Practice #1

A fast-food restaurant uses an automated filling machine to pour its soft drinks. The machine has different settings for small, medium, and large drink cups. According to the machine's manufacturer, when the large setting is chosen, the amount of liquid dispensed by the machine follows a Normal distribution with mean 27 ounces and standard deviation 0.8 ounces. When the medium setting is chosen, the amount of liquid dispensed follows a Normal distribution with mean 17 ounces and standard deviation 0.5 ounces. To test the manufacturer's claim, the restaurant manager measures the amount of liquid in a random sample of 25 cups filled with the medium setting and a separate random sample of 20 cups filled with the large setting. Let $\overline{x}_1 - \overline{x}_2$ be the difference in the sample mean amount of liquid under the two settings (large – medium).

1. What is the shape of the sampling distribution of $\overline{x}_1 - \overline{x}_2$? Why?

2. Find the mean and standard deviation of the sampling distribution.

Practice #1 Continued

3. Find the probability that $\overline{x}_1 - \overline{x}_2$ is more than 12 ounces. Show your work.

4. Based on your answer to Question 3, would you be surprised if the difference in the mean amount of liquid dispensed in the two samples was 12 ounces? Explain.

Practice #1 Answers

Normal because both X_1 and X_2 have Normal distributions.

$\mu_{\bar{x}_1 - \bar{x}_2} = 10$ ounces $\sigma_{\bar{x}_1 - \bar{x}_2} = 0.205$ ounces.

$P(\bar{x}_1 - \bar{x}_2 > 12) = P(z > 9.76) \approx 0.$

Since the probability in question 3 is so small, we would be very surprised to find a difference of 12 ounces.

Practice #2

How quickly do synthetic fabrics such as polyester decay in landfills? A researcher buried polyester strips in the soil for different lengths of time, then dug up the strips and measured the force required to break them. Breaking strength is easy to measure and is a good indicator of decay. Lower strength means the fabric has decayed.

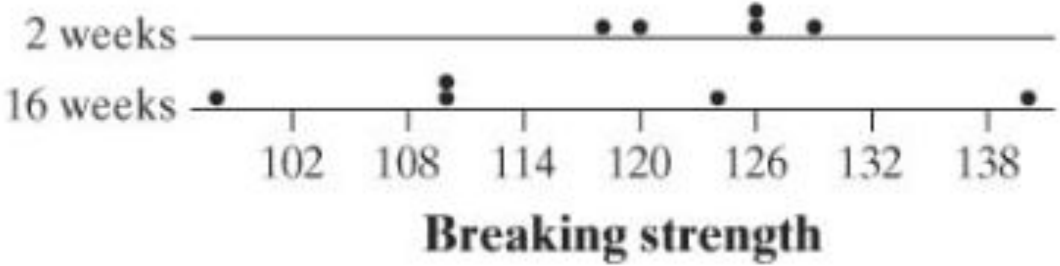
For one part of the study, the researcher buried 10 strips of polyester fabric in well-drained soil in the summer. The strips were randomly assigned to two groups: 5 of them were buried for 2 weeks and the other 5 were buried for 16 weeks. Here are the breaking strengths in pounds:[26]

Group 1 (2 weeks):	118	126	126	120	129
Group 2 (16 weeks):	124	98	110	140	110

Do the data give good evidence that polyester decays more in 16 weeks than in 2 weeks? Carry out an appropriate test to help answer this question.

Practice #2 Answer

State: We want to perform a test at the $\alpha = 0.05$ significance level of $H_0: \mu_1 - \mu_2 = 0$ versus $H_a: \mu_1 - \mu_2 > 0$, where μ_1 is the actual mean breaking strength at 2 weeks and μ_2 is the actual mean breaking strength at 16 weeks. **Plan:** Use a two-sample t test if the conditions are satisfied. *Random:* This was a randomized comparative experiment. *Normal:* Since n_1 and n_2 are less than 30, we examine the data. From the dotplot below, neither group displays strong skewness or any outliers. *Independent:* Due to the random assignment, these two groups of pieces of cloth can be viewed as independent. Also, knowing one piece of cloth's breaking strength gives no information about the breaking strength of another piece of cloth.

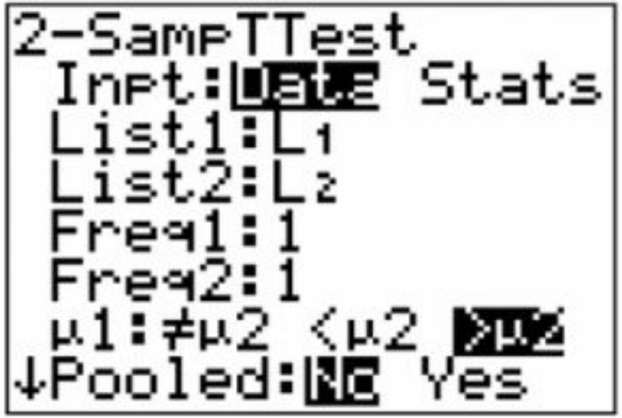


Practice #2 Answer Continued

Do: From the data, n_1 = 5, $\bar{x}_1 = 123.8$, s_1 = 4.60, n_2 = 5, $\bar{x}_2 = 116.4$, and s_2 = 16.09. Using the conservative df = 4, the test statistic is t = 0.989 and the P-value is $P(t > 0.989)$ = 0.1893. **Conclude:** Since the P-value is greater than 0.05, we fail to reject H_0. We do not have enough evidence to conclude that there is a difference in the actual mean breaking strength of polyester fabric that is buried for 2 weeks and fabric that is buried for 16 weeks.

Note: If you used your calculator to run a 2-SampTTest, you will get a very similar *t* value and p-value. The degree of freedom is df = 4.65

TI-83/84



Practice #3

The U.S. Department of Agriculture (USDA) conducted a survey to estimate the average price of wheat in July and in September of the same year. Independent random samples of wheat producers were selected for each of the two months. Here are summary statistics on the reported price of wheat from the selected producers, in dollars per bushel:[24]

Month	n	$\bar{x}$	s_x
July	90	\$2.95	\$0.22
September	45	\$3.61	\$0.19

Construct and interpret a 99% confidence interval for the difference in the mean wheat price in July and in September.

Practice #3 Answer

State: Our parameters are μ_1 = the true mean price of wheat in July and μ_2 = the true mean price of wheat in September. We want to estimate $\mu_1 - \mu_2$ at a 99% confidence level. **Plan:** Use a two-sample *t* interval for $\mu_1 - \mu_2$ if the conditions are satisfied. *Random:* Both samples were selected randomly. *Normal:* Both sample sizes were at least 30. *Independent:* Both samples are less than 10% of their respective populations (there are more than 900 wheat producers in July and 450 wheat producers in September). **Do:** From the data, $n_1 = 90$, $\bar{x}_1 = 2.95$, $s_1 = 0.22$, $n_2 = 45$, $\bar{x}_2 = 3.61$ = 3.61, and $s_2 = 0.19$. Using the conservative df = 44, the 99% confidence interval is $(2.95 - 3.61) \pm 2.692\sqrt{\frac{(0.22)^2}{90} + \frac{(0.19)^2}{45}} = (-0.759, -0.561)$. **conclude:** We are 99% confident that the interval from −0.759 to −0.561 captures the true difference in July and September. This suggests that the mean price of wheat in July is between $0.561 and $0.759 per bushel less than it is in September.

Practice #3 Answer Continued

Note: If you used your calculator to run a 2-SampTInterval, you will get a very similar interval. The degree of freedom is df = 100.45.
If you use your calculator, you must write out what you typed in and that includes the df!

More Practice

p.652-654 #37, 43 and 51

*Each problem is tied to a worked-out problem in the textbook and the solutions can be found in the back of the textbook.